and

$$A_2 = \frac{(1+r)^2\eta_s^2+\alpha_1^2+\xi^2\tan^2\theta_B(1-r)^2}{\eta_s^2\alpha_1^2+4\tan^2\theta_B\xi^2\eta_s^2+\tan^2\theta_B\xi^2\alpha_1^2}+\frac{1}{\alpha_2^2}. \quad (B8)$$

APPENDIX C
Monochromated source, expressions for σ and δ

The width parameter δ of the diffracted beam is given by

$$\delta^2 = 4A \Big/ \left[4A\left(\frac{a^2}{\omega_0^2}+\frac{1}{\eta_m^2}+\frac{1}{\eta_s^2}\right) - \left(\frac{2ab}{\omega_0^2}-\frac{2(M+2S)}{\eta_m^2}-\frac{2S}{\eta_s^2}\right)^2\right]. \quad (C1)$$

The rocking curve width parameter σ is found from the expression

$$\frac{1}{\sigma^2} = \frac{(A-S^2)}{\eta_s^2 A} - \frac{g^2}{\delta^2}. \quad (C2)$$

Where

$$A = \frac{1}{\xi^2}+\frac{b^2}{\omega_0^2}+\frac{(M+2S)^2}{\eta_m^2}+\frac{S^2}{\eta_s^2}. \quad (C3)$$

$$a = R_1 - R_0, \quad (C4)$$

and,

$$b = 2(M+2S)R_0 - 2SR_1. \quad (C5)$$

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Modified Ewald Construction for Neutrons Reflected by Moving Lattices

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A simple reciprocal lattice construction is presented which, in the case of neutrons reflected by moving lattices, permits Ewald construction directly in the laboratory frame without transforming the neutron velocity from the laboratory frame to the moving-crystal frame and back. Some special features and cases of the reflexion process – as seen in the laboratory frame – are discussed.

In recent years, an increased interest in neutron scattering by moving lattices has been shown (Lowde, 1957; Brockhouse, 1961; Shull & Gingrich, 1964; Meister, 1967; Shull, Morash & Rogers, 1968; Buras, Giebultowicz, Minor & Rajca, 1970). The experiments performed so far have shown easily measured changes in the reflexion process due to some kind of Doppler effect. It also seems plausible that the neutron diffraction effects observed in vibrating piezoelectric crystals by Galociova, Tichy, Zelenka, Michalec & Chalupa (1970) are strongly influenced by the Doppler effect (Buras, Giebultowicz, Minor & Rajca, to be published).

As proposed by Lowde (1957), and followed by Brockhouse (1961), Shull & Gingrich (1964), and Shull *et al.* (1968), the process of neutron diffraction by moving lattices is usually depicted in the reciprocal lattice space (Fig. 1) and consists of three steps: (1) the incident neutron velocity, $\mathbf{v}_i$, is transformed from the laboratory space to the moving-crystal space where it is equal to $\mathbf{u}_i$, (2) the reflected neutron velocity, $\mathbf{u}_r$, in the moving-crystal frame is found by means of Ewald construction, and (3) the reflected neutron velocity, $\mathbf{u}_r$, is transformed from the crystal moving frame to the laboratory frame, and the reflected neutron velocity, $\mathbf{v}_r = \overrightarrow{\mathrm{DE}}$, in the laboratory frame is finally obtained. This paper presents a simple procedure which directly permits reciprocal-lattice construction in the laboratory frame without laborious transformation from one frame of reference to another, and simplifies the calculations somewhat.

As seen in Fig. 1, $\overrightarrow{DE}$ equals $\overrightarrow{CB}$, and thus the velocity vectors $\mathbf{v}_i$ and $\mathbf{v}_r$ fulfill the condition (see triangle ABC):

$$\mathbf{v}_r - \mathbf{v}_i = \frac{h\tau}{m}, \tag{1}$$

which represents the momentum conservation law. A simple Galilean transformation of energies of the incident and reflected neutrons leads to the equation:

$$\frac{mv_r^2}{2} - \frac{mv_i^2}{2} = h\tau . \mathbf{V} = h\tau V \cos\beta , \tag{2}$$

which represents the energy conservation law. Thus, the Bragg reflexion of neutrons from a moving lattice is governed in the laboratory frame by two equations, (1) and (2).

In the crystal frame, the origins of the velocity vectors, $\mathbf{u}_i$ and $\mathbf{u}_r$ of the incident and reflected neutrons fulfilling the Bragg condition, lie on a straight line (marked as b in Fig. 1) which is the bisectrix of the $h\tau/m$ vector. This is the main feature of the well-known Ewald construction. As can be easily proved, in the laboratory frame the origins of the velocity vectors $\mathbf{v}_i$ and $\mathbf{v}_r$ of the incident and reflected neutrons fulfilling the Bragg condition, lie on a straight line (marked as b' in Fig. 1) parallel to b, but shifted by $-V\cos\beta$. This feature permits the reciprocal-lattice construction in the laboratory frame without transformation from one frame of reference to another. This type of reciprocal-lattice construction, depicting the reflexion process in the laboratory frame, we propose to call the modified Ewald construction.

As can be proved by simple mathematics, the conventional single Bragg law for a crystal at rest splits in the case of a moving lattice (as seen in the laboratory frame) into two laws: one for the incident neutron and another for the reflected one. We obtain (see Fig. 1) for the incident neutron:

$$2d \sin\theta_i = \lambda_i(1-f), \tag{3}$$

and for the reflected neutron:

$$2d \sin\theta_r = \lambda_r(1+f), \tag{4}$$

where θ_i and θ_r are the angle of incidence and of reflexion, λ_i and λ_r are the wavelengths of the incident and the reflected neutron in the laboratory frame, and the factor f is defined as

$$f = \frac{2mV\cos\beta}{h\tau}. \tag{5}$$

By means of the factor f, we can obtain simple and useful relations between θ_i, θ_r, λ_i and λ_r, *e.g.*

$$\frac{1}{\lambda_r^2} - \frac{1}{\lambda_i^2} = \frac{f}{d^2} \tag{6}$$

(d=interplanar spacing) and

$$\frac{(1+f)^2}{\sin^2\theta_r} - \frac{(1-f)^2}{\sin^2\theta_i} = 4f. \tag{7}$$

Equations (3)–(7) are exact and valid for all velocities V of the crystal and not only for V much lower than the neutron velocity v.

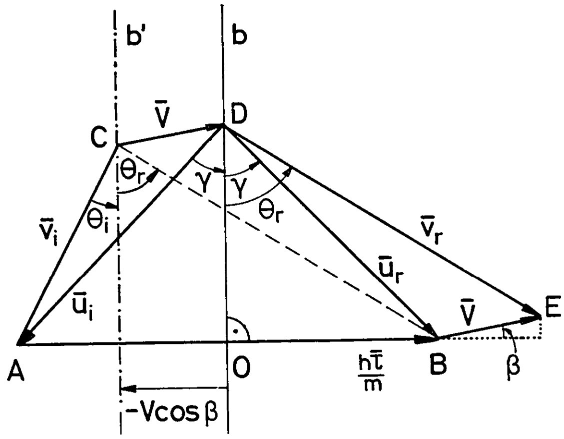

Fig. 1. Reciprocal lattice construction relating incident neutron velocity, $\mathbf{v}_i$, and scattered neutron velocity, $\mathbf{v}_r$, in the laboratory frame. $\mathbf{u}_i$ and $\mathbf{u}_r$ are the velocities of the incident and reflected neutron in the moving-crystal frame, $\mathbf{V}$ is the velocity of the crystal, θ_i and θ_r are the angles of incidence and reflexion in the laboratory frame, and γ is the angle of incidence (reflexion) sensed by the crystal. The other symbols have their usual meaning.

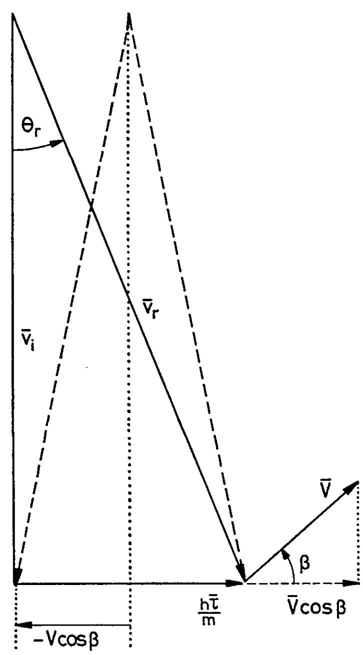

Fig. 2. Ewald construction in the case of a mica crystal ($d\simeq 10$ Å, $h\tau/m\simeq 400$ m.sec^{-1}) and incident neutron velocity v_i= 1000 m.sec^{-1} ($\lambda=4$ Å). Dashed line: crystal at rest; solid line: crystal moving with velocity V=200 m.sec^{-1} parallel to the $h\tau/m$ vector (f=1).

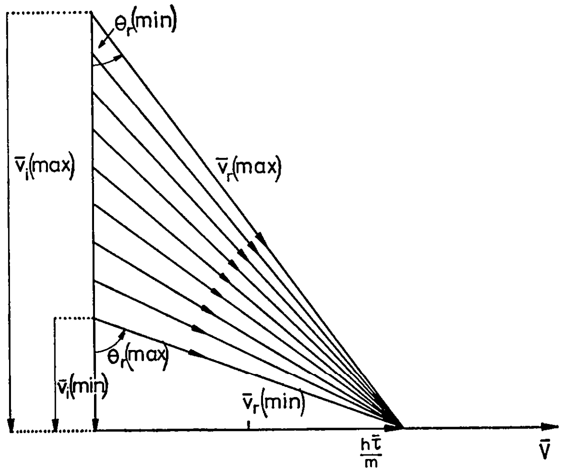

Fig. 3. Reflexion of a polychromatic neutron beam from a moving crystal in the case of $f=1$. $\mathbf{v}_i$(min) and $\mathbf{v}_i$(max) are the minimum and maximum velocities of the incident neutron; v_r(min) and v_r(max) are the minimum and maximum velocities of the reflected neutron, and θ_r(max) and θ_r(min) are the maximum and minimum angles of reflexion.

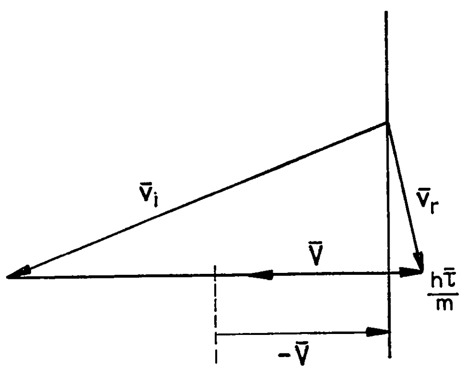

Fig. 4. The modified Ewald construction applied to the process of slowing-down of neutrons by means of reflexion from a moving crystal (see text.)

As seen from these equations, the factor f plays an important role in neutron diffraction by moving lattices as seen in the laboratory frame. This factor depends only on the reflexion in question (determined by the reciprocal lattice vector τ) and on the projection of the crystal velocity, $\mathbf{V}$, on the vector $\tau h/m$. It does not depend on the velocity of the incident neutron. The movement of the crystal has a particularly large influence on the diffraction process, as observed in the laboratory system for large interplanar spacings d. For example, in the case of a mica single crystal ($d \simeq 10$ Å) moving parallel to the reciprocal lattice vector with a velocity of 200 m.sec^{-1} (easily attainable in laboratory conditions), we have $f=1$. The modified Ewald construction in this case is shown in Fig. 2. The great difference between the Bragg reflexion of 4 Å neutrons ($v_i = 1000$ m.sec^{-1}) from the crystal at rest (dashed line) and from the moving crystal (solid line) is evident.

The case of $f=1$ may be of particular interest when the incident neutron beam is polychromatic. Fig. 3 presents the reciprocal lattice construction in the case of an incident beam of polychromatic neutrons whose velocities are in the range from $\mathbf{v}_i$(min) to $\mathbf{v}_i$(max). As seen from Fig. 3, the moving crystal scatters neutrons of different velocities (wavelengths) in different directions. It acts like an ordinary grating in the case of polychromatic light. However, the practical realization of such a moving-crystal neutron-velocity selector might be a bit difficult.

Fig. 4 shows the modified Ewald construction in the case when neutrons are slowed down by means of the Bragg reflexion from a moving single crystal. In the case of a mica crystal, moving with a velocity of about 200 m.sec^{-1} antiparallel to the $h\tau/m$ vector, incident neutrons with a velocity of 400 m.sec^{-1} ($\lambda = 10$ Å) can be, at least in principle, slowed down to very low velocities, and in this way supercold neutrons might be obtained.

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